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A Note on the Finite Taylor Series Applied in Laguerre Polynomials [1]

J. López-Bonilla, A. Lucas-Bravo

Abstract

The finite Taylor series with the emainder term is applied to Laguerre polynomials, showing thus the relationship between The talman'S identity and the fractional derivative for such polynomials.

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The finite Taylor series around the origin is given by the expansion [1]:

(1)

$$f(x) = f(0)+f^{(1)}(0)x+f^{(2)}(0)\frac{x^2}{2!}+\ldots+f^{(n-1)}(0)\frac{x^{n-1}}{(n-1)!}+\eta_n(x),\ n = 1, 2, \ldots$$

with the remainder term:

$$\eta_n(x) = \frac{1}{(n-1)!}\int_0^x (x-\xi)^{n-1} f^{(n)}(\xi)d\xi \equiv \frac{d^{-n}}{dx^{-n}} f^{(n)}(x), \quad (2)$$

where we have employed the notation for the derivative of Riemann-Liouville [2, 3]. If we take $f(x)$ as the associated Laguerre polynomials [4], $m \geq n$:

$$f(x) = L_m^{-n}(x) = \sum_{r=0}^{m} (-1)^r \left(\begin{array}{c} m-n \\ m-r \end{array} \right) \frac{x^r}{r!},\ m, n = 1, 2, \ldots \quad (3)$$

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then

(4) $$f^{(n)}(x) = (-1)^n L_{m-n}(x), \quad f^{(p)}(0) = 0, \quad p = 0, 1, \ldots, n-1,$$

therefore (1),. . . ,(4) imply the relation:

(5) $$\frac{d^{-n}}{dx^{-n}} L_{m-n}(x) = (-1)^n L_m^{-n}(x)$$

However, Talman [5] obtained the identity:

(6) $$L_m^{-n}(x) = (-1)^n \frac{(m-n)!}{m!} x^n L_{m-n}^n(x)$$

then (5) leads to the Abramowitz-Stegun [4] expression for the fractional derivative of Laguerre polynomials

(7) $$\frac{1}{(n-1)!} \int_0^x (x-\xi)^{n-1} L_{m-n}(\xi) d\xi = \frac{(m-n)!}{m!} x^n L_{m-n}^n(x).$$

or in the inverse order, if we accept the result of Abramowitz-Stegun then (5) gives us the Talman's identity.

We know the property:

(8) $$\frac{d^N}{dx^N} L_r^q(x) = (-1)^N L_{q-N}^{q+N}(x), \quad N = 0, 1, 2, \ldots$$

then from (5) we learn that (8) also is valid for $N = -1, -2, \ldots$

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Sección de Estudios de Posgrado e Investigación
Escuela Superior de Ingeniera Mecánica y Eléctrica
Instituto Politécnico Nacional
Edif. Z, Acc. 3-3er piso, Col. Lindavista, C.P. 07738 México, D.F.
E-mail: lopezbjl@hotmail.com
E-mail: andreslbravo@yahoo.com.mx