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# Students′ Difficulties with Application of Definite Integration [1]

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**Abstract**

The purpose of this study is to determine what special learning difficulties may be about the practices of definite integral. With this purpose, a test prepared by the researchers and consisting of 8 open-ended questions has been applied to 64 students from the Science Teaching department of Atatürk University Agri Educational Faculty during the general mathematics lesson. The data obtained from this study has been classified with their frequencies. In the direction of this obtained data, what kind of difficulties the students may have about the practices of determined integral have been pointed out.

**2000 Mathematical Subject Classification:** 97D50

**Key Words:** Learning difficulty, applications of definite integral, graphic drawing, the limits of definite integral

## 1 Introduction

To have a success in teaching of a certain subject, it is very important to determine what may be the special difficulties of students about the subject. It is especially more important for the subjects of mathematics.

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To develop and prepare learning environments providing efficient learning, teachers must be aware of the difficulties experienced by the students at learning mathematics (Yetkin, 2003). Contemporary view of education has encountered the teachers with the obligation and responsibility of choosing and applying the teaching methods and models which will provide learning at the maximum level (Yilmaz, 2001). To find this maximum level at learning, it is rather important that learning difficulties of a certain subject be determined beforehand.

In the process of learning and at the activities they take, the students are likely to have some learning difficulties and failures at some situations. It is the duty of the teacher as well as being a requirement of the modern teaching understanding to determine and eliminate such difficulties, and to help and guide to student during the learning process (Ersoy & Ardogan, 2003). Specifically in the field of mathematics, it is very much more important that teachers determine the learning difficulties of the students at the lesson.

The purpose of teaching mathematics is certainly to teach all the students successfully. However, it is known as a fact of life that although some of the students are successful at mathematics, a majority of them get a great difficulty at learning mathematics (Tall & Razali, 1993). Yudariah & Roselainy (2001a) pointed out that it is very significant to determine and eliminate the difficulties of students at learning mathematics immediately. For mathematics is more interdependently constructed and arranged subject according to other subjects, a concept can not be explained completely without giving its necessary pre-conditional concepts beforehand (Altun, 1998). Dikici and Işleyen, (2004) pointed out that it is very difficult for a student having problem at a certain subject to succeed in preceding subjects.

There have been various studies carried out to investigate students′ difficulties in Calculus. Some of the difficulties identified were poor understanding of basic concepts, inability to formulate problems mathematically, and

lack of mastery in algebraic, geometric and trigonometric skills (Yudariah & Roselainy, 2001b) Determining the learning difficulties at mathematics in their studies, Tall & Razali (1993) have been pointed out that students have difficulty at using the concepts and coordinating the operations. Also, while they state that the students perceiving operationally experience more problems than the students perceiving conceptually, Tall & Razali (1993), have given advices for eliminating the difficulties.

Tall (1993), have stated various studies have been enforcing for determining the difficulties at analysis; and some of these difficulties are that essential concepts be learnt inadequately by the students, the inadequacy of formulating the problems mathematically, and the lack of the algebraic, geometrical, and trigonometrically abilities of students. Researching into the difficulties encountered by the students in the concept of multiple algebraic, Yudariah & Roselainy, (2001b) formed into 4 main categories. These are:

I. Inspection of zones and spaces

II. Interpreting the graphics

III. Algorithmically mistakes

IV. Algebraic operation mistakes

Durmuş(2004), in his study purposing to define the subjects perceived as being difficult at mathematics lesson in high school curriculum, and to bring into light the causes laying under these difficulties, established the difficulty index of all the subjects in mathematics lesson at the intermediate education curriculum with a likert type of investigation. Durmuş(2004), at the results of interviews with the students, have stated that the lack of motivation and the abstraction of concepts are two important points for the reason for these difficulties.

Erbaş & Ersoy, (2002), have shown that a group of high school students from different schools have success and relatively difficulties, mistakes and possible conceptual misperceptions in solving equation. In his study,

Weber (2001) stated the difficulties encountered by university students in forming the proof of an expression in mathematics. Gonzales-Martin & Camacho (2004) expressed that that there are students who have difficulties in articulating the different systems of representation, and have problems in connecting and relating this knowledge as a generalization of previous concepts, such as definite integrals, series and sequences.

Since it is one of the most important subjects of analysis, the application of definite integral subject gains more and more significance. It is very important for a mathematics practitioner to know the difficulties in teaching this subject. For this reason, in this study, it is purposed to determine what special difficulties may be encountered by the students when learning the subject of definite integral and its applications.

# 2 Methodology

## 2.1 Participants

This study has been made over 64 students from second class of Science Teaching Department at Atatürk University Agri Educational Faculty in first term of 2005-2006 Education Semester.

## 2.2 Procedure and data analysis

In this study for determining what special learning difficulties about the subject of application of definite integral may be, a test of consisting of 8 open-ended question prepared by the researchers has been applied to students who participated the research immediately after they are given the subject of definite integral and its applications (by the teacher of the lesson) in general mathematics lesson at Atatürk University Agri Educational Faculty from the department of Science Teaching. The data obtained from

this research has been classified with their frequencies.

# 3 Findings

We start by giving a brief explanation of the 8 open-ended questions we used in this research. These questions involve drawing the graphic of a function, finding the region and calculating the definite integral. The answers of the students to these 8 open-ended questions are analyzed according to the classification in Table 1;

| *Questions* | *Description* | *Abilities assessed* |
|---|---|---|
| *1, 2, 3, 4* | The accounting of the area between given functions and axis with their equation | a. Constructing the region<br>b. Defining the limits of integral<br>c. The accurate determination of the function to be accounted of integral<br>d. The accounting of integral |
| *5, 6, 7, 8* | The accounting of a region given in a graphic with integral | a. The determination of the limits of integral<br>b. The accurate determination of the function whose integral will be accounted<br>c. The accounting of integral |

Table 1: Description of questions and abilities assessed

The first 4 of questions used in the research asked with the purpose of calculating of the area between the functions with its equation and axis by using integral. By taking into consideration the classification in Table 1, the dispersion of the responses given by the students is stated in Table 2;

| *Questions* | *1* | | | | *2* | | | | *3* | | | | *4* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Abilities* | a | b | c | d | a | b | c | d | a | b | c | d | a | b | c | d |
| *Correct* | 48 | 46 | 32 | 42 | 38 | 36 | 34 | 56 | 33 | 31 | 10 | 53 | 28 | 16 | 13 | 35 |
| *Incorrect* | 14 | 16 | 28 | 14 | 24 | 26 | 28 | 6 | 28 | 28 | 49 | 5 | 28 | 35 | 35 | 4 |
| *No answer* | 2 | 2 | 4 | 8 | 2 | 2 | 2 | 2 | 3 | 5 | 5 | 6 | 8 | 13 | 16 | 25 |

Table 2: Distribution of answers to test items 1, 2, 3, 4

As viewed in the Table 2, in determining the region whose area will be accounted with integral; the students do not have much difficulty in drawing the functions of polynomial type that are already accustomed by them; on the other hand, they get difficulty in drawing the graphics of functions that of not polynomial type. At question 1; 19 students, and at the question 3; 25 students did not pay attention whether the formulated area is under the x-axis, when determining the function whose integral to be taken. The response of one of these students is given in Figure 1.

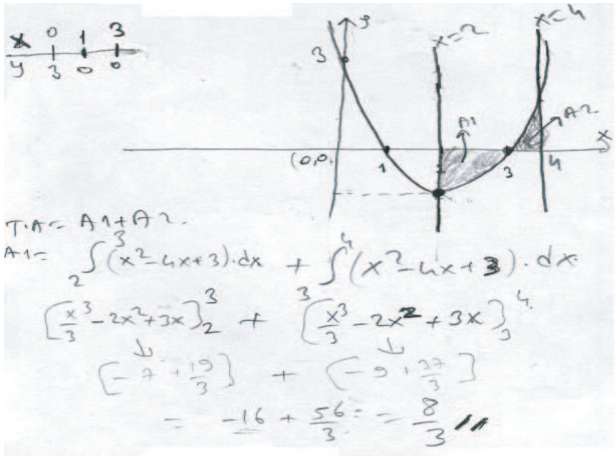



Figure 1: The student′s answer to question 1

While 6 of the students at question 2 and 16 students at question 3 formed the region accurately whose area will be accounted, they made mistake because they stated the asked area at only one integral when determining the limits of the integral. In the Figure 2, the response of a student

who responded in that way is given;

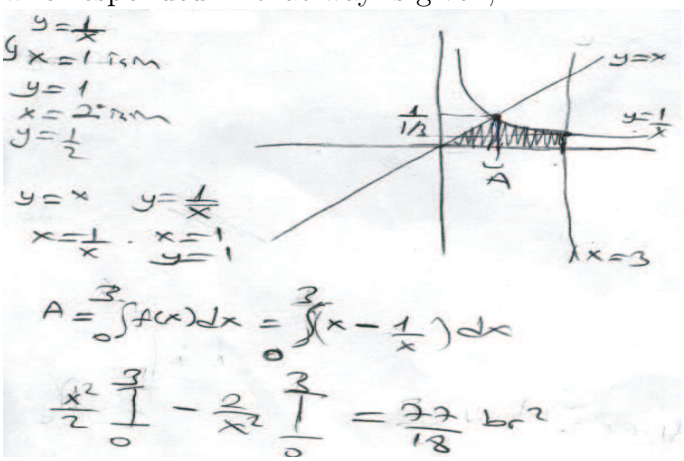


Figure 2: The student's answer to question 3

At the question 2, when forming the region that the area is accounted, 17 students have made mistake for not taking into consideration the axises. In other words; without taking into consideration the x-axis in the question, these students did only take the area between the curves of and . One of these responses is given in Figure 3.

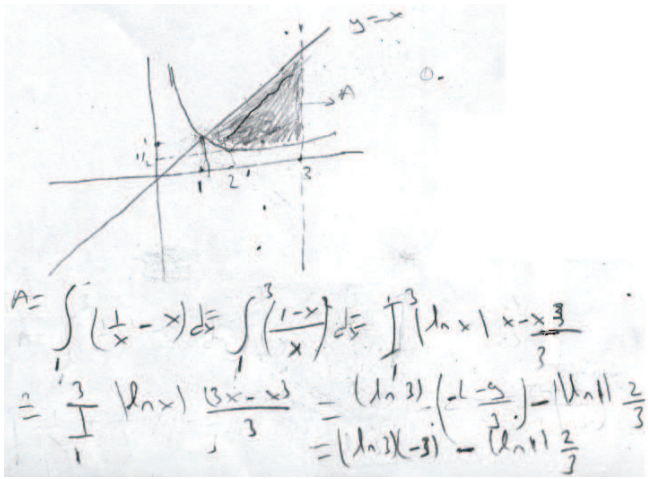


Figure 3: The student's answer to question 2

| *Questions* | *5* | | | *6* | | | *7* | | | *8* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Abilities* | a | b | c | a | b | c | a | b | c | a | b | c |
| *Correct* | 51 | 49 | 51 | 58 | 55 | 33 | 40 | 39 | 51 | 55 | 55 | 58 |
| *Incorrect* | 12 | 14 | 11 | 5 | 8 | 17 | 22 | 23 | 9 | 5 | 5 | 2 |
| *No answer* | 1 | 1 | 2 | 1 | 1 | 14 | 2 | 2 | 4 | 4 | 4 | 4 |

Table 3: Distribution of answers to test items 5, 6, 7, 8

When Table 3 and Table 2 is compared, it is observed that the rate of success of students at 5, 6, 7, and 8. questions is much more than the rate of success at 1,2,3, and 4. questions. At this result, it has been efficient to give the region whose area will be accounted within the question. 12 of the students participating the research made mistake since they needed to state the given area at one integral when determining the limits of the integral, same as the mistake at the question 2 and 3. In the Figure 4, the response of a student who responded in this way is given;

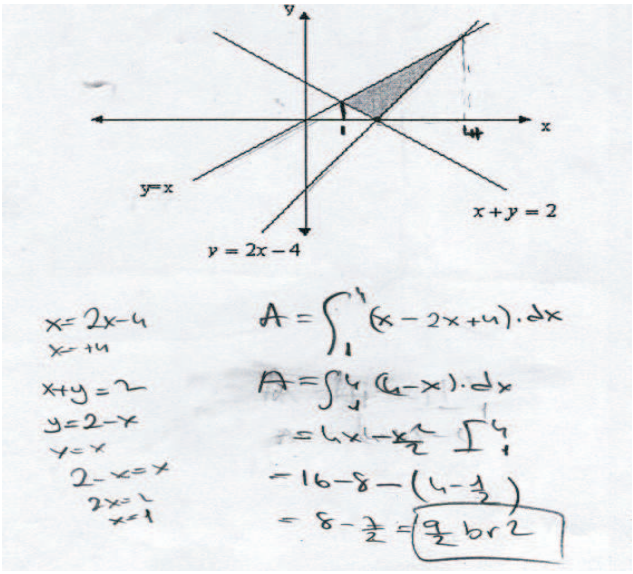



Figure 4: The student′s answer to question 5

In this research which was executed with the purpose of determining what special difficulties the students may have when learning definite inte-

gral; it is appeared that when forming the area whose space to be accounted, the students have difficulty in drawing the graphics of functions except of polynomial type. For this reason, this problem of the students has affected the rate of achievement in the subject of application of definite integral negatively.

At the questions about calculating the space between the functions together with equations and axis, although the students have difficulty in forming the asked area, determining the limits of the integral and determining the area whose integral to be taken, they do not have much difficulty in calculating the integral of the function they determined.

Although one part of the students participated the research have given accurately the asked area and limits in the question, they made mistake by not taking into consideration that this area be under the x-axis.

While some of the students in the example formed the area whose space is to be calculated accurately, they made mistake by needing to give the requiring area under one integral. We can say that this mistake appears because the definition of the area between two curves is misconceived by the students. Since this definition is not learnt at a conceptual basis, or in other words it is memorized, and with the direction of definition at the question; the students tried to form expression in the asked space of the question. Some of the students also took only the area between curves without taking into consideration the axis when forming the area whose space to be calculated. As a result, the area whose space be calculated could not be calculated accurately. According to this result, it can be said that the students have not the adequate knowledge of concepts about the subject. This conclusion has been parallel to the study of Sabelle & Redish (1995), in which they pointed out that "The main difficulty of students is to learn the concepts in the given subjects, not to learn algorithmically operations. However; mostly of the American students; nearly from all over the world, the mathematical experiences of the students are nothing but

calculations and operations. Mathematical subject have a more strongly sequenced structure. Fort his reason, a concept can not be explained accurately without giving its pre-conditional concepts beforehand. One of the pre-conditional concepts of definite integral subject is to draw the graphic of a given function. It can not be expected from the students having difficulty in drawing the graphic of a function to be successful at the applications of definite integral. Because of this reason, before explaining a new subject, the pre-conditional concepts of the subject should be clarified, and then the possible difficulties about these concepts should be determined and eliminated.

It is very significant at mathematics that strong conceptual basis be articulated. Because of this reason, the conceptual basis about the applications of definite integral subject should be constructed properly.

APPENDIX

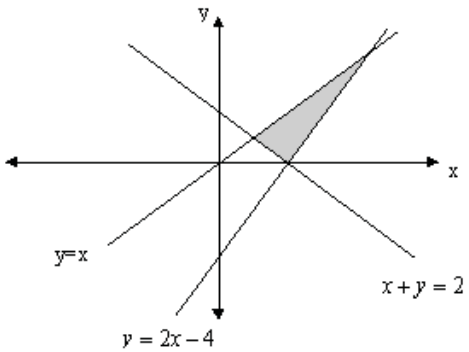
1. Find the area bounded the parabola $y = x^2 - 4x + 3$, the lines $x = 2$ and $x = 4$ and x-axis.

2. Find the area bounded the curves $y = -\sqrt{x}$, $y = -2\sqrt{x}$ and the line $y = -x$.

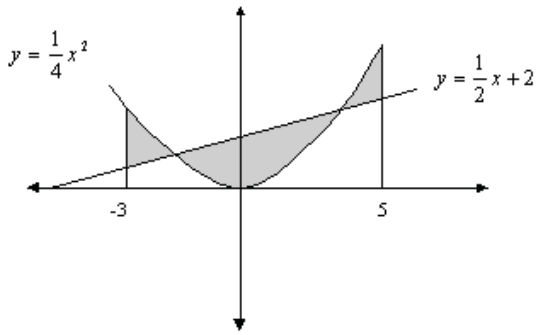
3. Find the area between the curve $y = \dfrac{1}{x}$ the lines $y = x$ and $x = 3$ and x-axis.

4. Find the area between the curve $y = \sqrt{1 = x^2}$, line $y = x$ and y-axis.

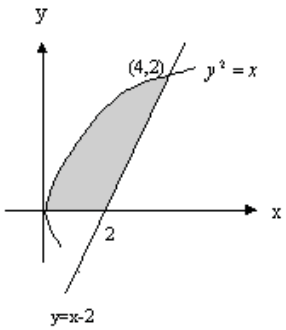
5. Find the area of the shaded region in the following figure.



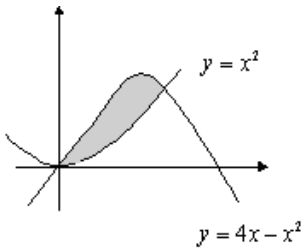
6. Find the area of the shaded region in the following figure.




7. Find the area of the shaded region in the following figure.




8. Find the area of the shaded region in the following figure.

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